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LOG PEARSON TYPE 3 DISTRIBUTION:  
TABLES OF QUANTILES

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## LOG PEARSON TYPE 3 DISTRIBUTION

### BACKGROUND

Log Pearson type 3 (LP) distribution is extensively used in hydrologic frequency analysis. The traditional fitting procedure consists of transformation of natural data into logarithms and fitting logarithmic data to a Pearson type 3 (P) distribution by the method of moments. In this method (logarithmic moments method--LGMO) the quantiles of LP distribution, i.e., flood flows, low flows, etc. for different return periods, are obtained on the basis of mean, variance, and skewness coefficient of logarithmic data from the tables provided in Ref. 2. Ref. 2 also describes the use of a regional skewness coefficient when the data analyzed are flood flows.

Other methods of fitting LP distribution to hydrologic data have recently been introduced [Bobee (1), Rao (3, 4)]. In Bobee's method quantiles of LP distribution are obtained on the basis of the moments of real data (as opposed to logarithmic data); however, Bobee's method uses the origin moments of data. By a study conducted at this District, the writer has developed the following other methods:

1. Method of Real Moments (RLMO): This is a variation of Bobee's method. In this method the quantiles of LP distribution are obtained on the basis of the more familiar statistical parameters, mean, variance, and skewness coefficient (of real data) instead of the origin moments used by Bobee.
2. Methods of Mixed Moments: In these methods the use of skewness coefficient (the estimate of which is generally biased for small sample sizes) is eliminated. Two variations of these methods are available.

I. Method of Mixed Moments - I (MXM1): In this method the quantiles of LP distribution are estimated on the basis of the mean and variance of real data and the mean of logarithmic data.

II. Method of Mixed Moments - II (MXM2): In this method the quantiles are estimated on the basis of the mean and variance of logarithmic data and the mean of real data.

Details of RLMO, MXM1 and MXM2 methods are available in References 3 and 4.

#### EVALUATION OF LOG PEARSON QUANTILES

Mathematical evaluation of LP quantiles is quite involved; especially, their evaluation by desk calculations is well nigh impossible. They are evaluated by a computer program or generalized tables specially derived for the purpose. A computer program was developed by the writer to obtain solutions of LP distribution by LGMO, RLMO, MXM1 and MXM2 using different mathematical and statistical procedures described in References 3 and 4. Exhibit 1 shows typical computer output from the program. [Maximum likelihood (MXLK) method mentioned in Exhibit 1 is a different procedure, details of which will be presented in a future publication.] Generalized tables were also derived in terms of the dimensionless variate K (see Ref. 3), to obtain LP quantiles for RLMO and MXM1 without using the computer program. These tables are included in this memorandum (pp. 1 to 17) and their use is explained below. (To obtain LP quantiles for MXM2 the tables available in Ref. 2 may be used as explained in Ref. 4.)

## USE OF THE TABLES

### I. Method of Real Moments (RLMO):

The following steps may be followed:

1. Obtain mean,  $\bar{X}$ , of real data ( $x_1, x_2, \dots, x_i \dots, x_N$ ) by

$$\bar{X} = \frac{1}{N} \sum_{i=1}^N x_i \quad \dots \dots \dots \dots \dots \dots \dots \quad (1)$$

(N = sample size)

2. Convert data into dimensionless variables by  $K_i = x_i / \bar{X}$ .
3. Calculate the unbiased estimates of variance,  $S_k^2$ , and skewness coefficient,  $CS_k$ , from dimensionless data by equations,

$$\text{Variance, } S_k^2 = \frac{1}{(N - 1)} \sum_{i=1}^N (K_i - \bar{K})^2 \quad \dots \dots \dots \dots \quad (2)$$

$$\text{Skewness Coefficient, } CS_k = \frac{N}{S_k^3 (N - 1) (N - 2)} \sum_{i=1}^N (K_i - \bar{K})^3 \quad \dots \dots \dots \quad (3)$$

Note that  $\bar{K}$ , the mean of dimensionless data = 1.0.

4. Enter the tables and obtain K values of interest. Use interpolation, if necessary.
5. Multiply K values obtained in Step 4 by  $\bar{X}$  to obtain final results.

Example: For the sample in Exhibit I (annual flood flows), assume that

$\bar{X}$ ,  $S_k^2$  and  $CS_k$  were calculated as 3392.5 cfs, 0.543, and 1.392, respectively. Determine 100-year flood flow.

By interpolation, the tables (page 4) give  $K = 3.420$ . Therefore, 100-year flood flow =  $3.420 \times 3392.5 = 11,600$  cfs (rounded).

II. Method of Mixed Moments - I (Mxm1): In this method, the value of  $CS_k$  is selected on the basis of  $\bar{Y}$ , the mean of logarithmic data, instead of calculating by Eq. 3. Follow the following steps:

1. Perform Steps 1 and 2 of RLMO.
2. Obtain logarithmic mean,  $\bar{Y}$ , of dimensionless data by

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^N \ln K_i \quad \dots \dots \dots \dots \dots \dots \dots \quad (4)$$

3. Calculate  $S_k^2$  by Eq. 2.
4. Enter Exhibit II and determine the value of  $CS_k$ , (Use interpolation, if necessary) for  $\bar{Y}$  and  $S_k^2$  calculated in the above steps.
5. Perform Steps 4 and 5 of RLMO.

Example: For the sample in Exhibit I,  $\bar{X} = 3392.5$  cfs,  $\bar{Y} = -0.2278$  and

$$S_k^2 = 0.5428. \text{ Determine 100-year flood flow.}$$

Exhibit II gives, by interpolation,  $CS_k = 2.338$ . Tables of  $K$  values (pages 6-7) give, by interpolation,  $K = 3.6895$  for 100-year return period. Thus, 100-year flood value =  $3.6895 \times 3392.5 = 12,500$  cfs (rounded).

#### REFERENCES

1. Bobee, B., "The Log Pearson Type 3 Distribution and Its Application in Hydrology," Water Resources Research, Vol. 11, No. 5, October 1975, pp. 681-689.
2. "Guidelines for Determining Flood Flow Frequency," Bulletin No. 17, Water Resources Council, Washington, D. C., 1976.
3. Rao, D. V., "Log Pearson Type 3 Distribution: A Generalized Evaluation," Journal of the Hydraulics Division, ASCE, Vol. 106, No. HY5, May 1980, pp. 853-872.
4. Rao, D. V., "Log Pearson Type 3 Distribution: Method of Mixed Moments," Journal of the Hydraulics Division, ASCE, Vol. 106, No. HY6, June 1980, pp. 999-1019.

--- FREQUENCY ANALYSIS BY LOG PEARSON TYPE III DISTRIBUTION ---

\*\*\*\*\* ECONLOCKHATCHEE RIVER NR CHULUOTA, FLA. -ANNUAL PEAK FLOWS(1936-75) IN CFS

1760.00	1760.00	1490.00	2380.00	950.00	2070.00	1360.00	1960.00	6100.00	8650.00
1190.00	4240.00	9570.00	3610.00	7210.00	1960.00	2590.00	3980.00	1740.00	1330.00
8580.00	3150.00	2290.00	4120.00	10100.00	3930.00	1810.00	3520.00	5290.00	1290.00
2230.00	2150.00	6050.00	4230.00	2070.00	1340.00	1500.00	1490.00	3750.00	810.00

\*\*\* SAMPLE SIZE= 40 MEAN OF DATA= 3392.50

RMM REAL MOMENTS METHOD:-

MEAN, VARIANCE AND SKEW OF D-LESS DATA: 1.00000 0.54276 1.39154  
 ESTIMATES OF PARAMETERS A, B, & C: -0.36410E 01 0.91203E 01 0.22132E 01  
 ESTIMATES OF LOG MEAN, VAR AND SKEW: -0.29165 0.68796 -0.66226

RPM LOG MOMENTS METHOD:-

MEAN VAR AND SKEW OF LOG DATA: -0.22780 0.44681 0.39069  
 ESTIMATES OF PARAMETERS A, B, & C: 0.76585E 01 0.26206E 02 -0.36497E 01  
 ESTIMATES OF MEAN, VAR AND SKEW FOR REAL DATA: 1.01732 0.84682 5.59157

RPM MIXED MOMENTS-I METHOD:-

MEAN, VARIANCE AND SKEW OF D-LESS DATA FIT: 1.00000 0.54276 2.33493  
 ESTIMATES OF PARAMETERS A, B, & C: -0.26020E 02 0.31633E 03 0.11929E 02  
 ESTIMATES OF LOG MEAN, VAR AND SKEW: -0.22780 0.46722 -0.11245

RPM MIXED MOMENTS-II METHOD:-

MEAN VAR AND SKEW OF LOG DATA FIT: -0.22780 0.44681 0.08643  
 ESTIMATES OF PARAMETERS A, B, & C: 0.34620E 02 0.53552E 03 -0.15696E 02  
 ESTIMATES OF MEAN, VAR AND SKEW FOR REAL DATA: 1.00000 0.60639 3.07136

PLM MAXIMUM LIKELIHOOD METHOD:-

FINAL ESTIMATES OF A, B, C, MEAN, VAR, & SKEW-  
 -0.11E 04 0.49E 06 0.46E 03 0.9900 0.5342 2.6097  
 0.39E 01 0.70E 01 -0.20E 01 1.0483 1.5250 28.5667  
 SELECTED A,B,C: -0.10584E 04 0.48827E 06 0.46110E 03  
 ESTIMATES OF LOG MEAN, VAR AND SKEW: -0.22773 0.43587 -0.00286

NUMERICAL VALUES OF VARIABLE K FOR SELECTED FREQUENCIES  
 LOG PEARSON TYPE III DISTRIBUTION  
 NON-EXCEEDANCE PROBABILITY

	0.005	0.010	0.020	0.040	0.100	0.200	0.500	0.800	0.900	0.960	0.980	0.990	0.995	0.998	0.999
	RECURRANCE INTERVAL IN YEARS FOR THE ANNUAL LARGEST EVENTS														
	1.005	1.010	1.020	1.042	1.111	1.250	2.	5.	10.	25.	50.	100.	200.	500.	1000.
RECURRANCE INTERVAL IN YEARS FOR THE ANNUAL SMALLEST EVENTS															
METHOD (1)	200.	100.	50.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
RLMO	0.053	0.073	0.103	0.147	0.248	0.387	0.818	1.521	2.004	2.600	3.022	3.420	3.796	4.261	4.589
LGM0	0.182	0.204	0.233	0.272	0.349	0.450	0.762	1.375	1.919	2.793	3.599	4.554	5.684	7.496	9.150
MXM1	0.127	0.153	0.188	0.234	0.329	0.450	0.807	1.420	1.896	2.565	3.109	3.690	4.309	5.189	5.903
MXM2	0.150	0.175	0.208	0.252	0.340	0.453	0.789	1.393	1.887	2.617	3.241	3.934	4.703	5.849	6.824
MXLK	0.145	0.171	0.205	0.251	0.342	0.457	0.797	1.388	1.856	2.528	3.087	3.694	4.354	5.313	6.109

\*\*\* VARIATE VALUES \*\*\*

CDF	T-LOS	T-PMS	BY RLMO	BY MXM1	BY MXLK	BY LGMO	BY MXM2
0.005	200.	1.	179.99	432.13	492.38	616.80	509.79
0.010	100.	1.	248.92	520.67	580.74	692.58	595.32
0.020	50.	1.	349.43	637.05	695.51	790.28	706.23
0.040	25.	1.	500.27	795.43	849.91	921.38	855.30
0.100	10.	1.	840.08	1116.26	1158.96	1185.27	1154.38
0.200	5.	1.	1312.18	1525.88	1550.07	1525.50	1535.12
0.500	2.	2.	2775.58	2736.12	2702.42	2586.61	2675.40
0.800	1.	5.	5159.31	4818.37	4709.35	4664.29	4727.12
0.900	1.	10.	6799.24	6430.80	6294.93	6510.66	6400.17
0.960	1.	25.	8819.71	8701.87	8576.28	9476.05	8878.03
0.980	1.	50.	10250.83	10548.52	10472.42	12208.77	10993.81
0.990	1.	100.	11402.71	12518.32	12532.76	15449.23	13344.46
0.995	1.	200.	12879.09	14617.97	14770.35	19282.87	15954.93
0.998	1.	500.	14455.33	17603.64	18024.89	25429.86	19844.07
0.999	1.	1000.	15567.32	20027.30	20724.69	31040.99	23149.58

Final Results:  
 Peak Flows in cfs

↑ Return Period, in years, for flood flows  
 ↑ Return Period, in years, for low flows  
 Nonexceedance Probability

Logarithmic Mean ( $\bar{Y}$ ) Values for Different Variance ( $S_k^2$ ) and Skewness Coefficients ( $CS_k$ ) of the Dimensionless Variate, K

VARIANCE OF K	SKEWNESS COEFFICIENT												
	-1.0 (1)	-0.8 (2)	-0.6 (3)	-0.4 (4)	-0.2 (5)	0.0 (6)	0.2 (7)	0.4 (8)	0.6 (9)	0.8 (10)	1.0 (11)	1.2 (12)	1.4 (13)
0.03	-0.0181	-0.0175	-0.0170	-0.0165	-0.0161	-0.0157	-0.0153	-0.0150	-0.0147	-0.0144	-0.0141	-0.0138	-0.0136
0.04	-0.0253	-0.0243	-0.0234	-0.0226	-0.0219	-0.0213	-0.0207	-0.0201	-0.0196	-0.0192	-0.0188	-0.0184	-0.0180
0.05	-0.0331	-0.0316	-0.0302	-0.0290	-0.0279	-0.0270	-0.0261	-0.0254	-0.0247	-0.0240	-0.0234	-0.0229	-0.0224
0.06	-0.0416	-0.0393	-0.0374	-0.0357	-0.0342	-0.0329	-0.0317	-0.0307	-0.0298	-0.0289	-0.0282	-0.0275	-0.0268
0.07	-0.0509	-0.0477	-0.0450	-0.0427	-0.0407	-0.0390	-0.0375	-0.0362	-0.0350	-0.0339	-0.0329	-0.0321	-0.0313
0.08	-0.0610	-0.0566	-0.0531	-0.0501	-0.0475	-0.0453	-0.0434	-0.0417	-0.0402	-0.0389	-0.0377	-0.0367	-0.0357
0.09	-0.0720	-0.0663	-0.0616	-0.0578	-0.0545	-0.0518	-0.0494	-0.0474	-0.0456	-0.0440	-0.0426	-0.0413	-0.0402
0.10	-0.0842	-0.0767	-0.0707	-0.0659	-0.0619	-0.0585	-0.0556	-0.0532	-0.0510	-0.0492	-0.0475	-0.0460	-0.0447
0.15	-0.1681	-0.1433	-0.1259	-0.1131	-0.1033	-0.0956	-0.0893	-0.0841	-0.0798	-0.0761	-0.0729	-0.0701	-0.0677
0.20	-0.3261	-0.2491	-0.2043	-0.1751	-0.1546	-0.1395	-0.1279	-0.1187	-0.1112	-0.1051	-0.0999	-0.0955	-0.0917
0.25	-0.7117	-0.4387	-0.3226	-0.2591	-0.2194	-0.1922	-0.1724	-0.1575	-0.1458	-0.1364	-0.1287	-0.1222	-0.1168
0.30	-2.4547	-0.8574	-0.5182	-0.3783	-0.3030	-0.2561	-0.2243	-0.2013	-0.1839	-0.1703	-0.1594	-0.1505	-0.1431
0.35	-177.8600	-2.2803	-0.8925	-0.5584	-0.4144	-0.3351	-0.2852	-0.2510	-0.2261	-0.2072	-0.1924	-0.1805	-0.1706
0.40	-20.7087	-1.8175	-0.8563	-0.5689	-0.4347	-0.3576	-0.3077	-0.2729	-0.2473	-0.2277	-0.2122	-0.1996	
0.45		-5.7979	-1.4235	-0.7952	-0.5631	-0.4445	-0.3729	-0.3252	-0.2911	-0.2656	-0.2458	-0.2300	
0.50		-124.9000	-2.7958	-1.1521	-0.7343	-0.5506	-0.4484	-0.3836	-0.3390	-0.3064	-0.2816	-0.2621	
0.55			-8.1761	-1.7801	-0.9716	-0.6825	-0.5367	-0.4494	-0.3915	-0.3503	-0.3196	-0.2958	
0.60			-92.3732	-3.0910	-1.3185	-0.8500	-0.6410	-0.5239	-0.4493	-0.3978	-0.3600	-0.3313	
0.65				-6.7779	-1.8628	-1.0684	-0.7657	-0.6087	-0.5131	-0.4491	-0.4031	-0.3687	
0.70				-26.1666	-2.8086	-1.3630	-0.9170	-0.7060	-0.5839	-0.5047	-0.4492	-0.4082	
0.75					-4.7237	-1.7775	-1.1037	-0.8184	-0.6627	-0.5651	-0.4983	-0.4498	
0.80					-9.7148	-2.3939	-1.3387	-0.9496	-0.7508	-0.6309	-0.5510	-0.4939	
0.85					-30.6443	-3.3816	-1.6417	-1.1042	-0.8499	-0.7029	-0.6074	-0.5406	
0.90						-5.1387	-2.0443	-1.2888	-0.9620	-0.7817	-0.6680	-0.5900	
0.95						-8.7888	-2.5992	-1.5120	-1.0895	-0.8684	-0.7333	-0.6423	
1.00						-18.5103	-3.4002	-1.7864	-1.2357	-0.9642	-0.8037	-0.6980	
1.25							-42.7478	-5.0548	-2.4443	-1.6368	-1.2556	-1.0357	
1.50								-44.4770	-6.0138	-2.9429	-1.9743	-1.5143	
1.75									-30.2336	-6.1837	-3.2464	-2.2314	
2.00										-19.4246	-5.9117	-3.3929	
2.25											-13.4035	-5.5003	
2.50												-10.0579	





## TABLES OF K VALUES FOR SELECTED FREQUENCIES

These tables were derived by a computer program specifically developed for the purpose using the theories described in Ref. 3. Thus, the tables are deemed accurate. However, any inaccuracies noticed may please be brought to the attention of the author.

For each skew value, the tables are terminated when the form of LP type 3 density becomes U-shape or when the variance reaches a value of 10.0.

































